# CSC413/2516 Lecture 11: Q-Learning & the Game of Go

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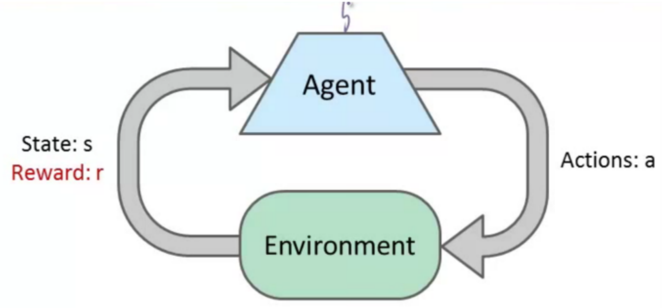
# Overview

- Second lecture on reinforcement learning
  - Previously, we have seen how to optimize a policy directly
- Today: Q-learning
  - Learn an action-value function that predicts future returns
- Case study: AlphaGo uses both a policy network and a value network

# Overview

- Agent interacts with an environment, which we treat as a black box
- Your RL code accesses it only through an API since it's external to the agent
  - I.e., you're not "allowed" to inspect the transition probabilities, reward distributions, etc.



# Recap: Markov Decision Processes

- The environment is represented as a Markov decision process (MDP) $\mathcal{M}$.
- Markov assumption: all relevant information is encapsulated in the current state
- Components of an MDP:
  - initial state distribution $p(\mathrm{s}_0)$
  - transition distribution $p(\mathrm{s}_{t+1} \mid \mathrm{s}_t, \mathrm{a}_t)$
  - reward function $r(\mathrm{s}_t, \mathrm{a}_t)$
- policy $\pi_{\boldsymbol{\theta}}(\mathrm{a}_t \mid \mathrm{s}_t)$ parameterized by $\boldsymbol{\theta}$
- Assume a fully observable environment, i.e. $\mathrm{s}_t$ can be observed directly

# Finite and Infinite Horizon

- Last time: finite horizon MDPs
  - Fixed number of steps $T$ per episode
  - Maximize expected return $R = \mathbb{E}_{p(\tau)}[r(\tau)]$
- Now: more convenient to assume infinite horizon
  - We can't sum infinitely many rewards, so we need to discount them: \$100 a year from now is worth less than \$100 today
  - Discounted return

$$G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots$$

  - Want to choose an action to maximize expected discounted return
  - The parameter $\gamma < 1$ is called the discount factor
    - small $\gamma$ = myopic
    - large $\gamma$ = farsighted

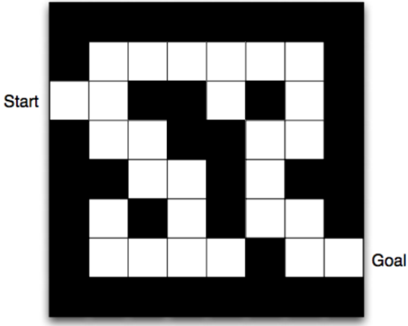
# Value Function

- Value function $V^{\pi}(\mathrm{s})$ of a state s under policy $\pi$: the expected discounted return if we start in s and follow $\pi$

$$
\begin{aligned}
V^{\pi}(\mathrm{s}) &= \mathbb{E}[G_t \,|\, \mathrm{s}_t = \mathrm{s}] \\
&= \mathbb{E}\left[\sum_{i=0}^{\infty} \gamma^i r_{t+i} \,|\, \mathrm{s}_t = \mathrm{s}\right]
\end{aligned}
$$

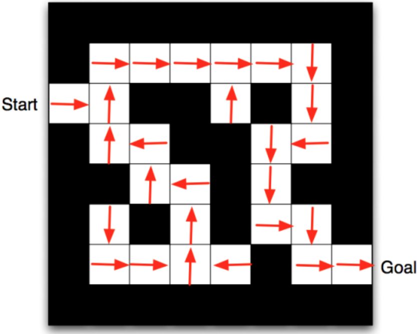
- Computing the value function is generally impractical, but we can try to approximate (learn) it
- The benefit is credit assignment: see directly how an action affects future returns rather than wait for rollouts

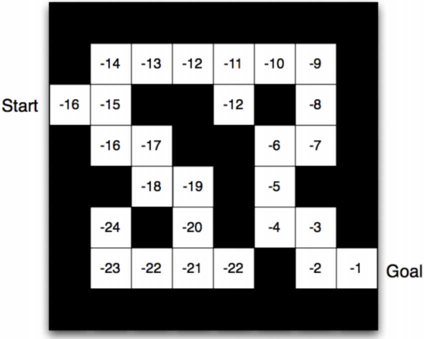
# Value Function




- Rewards: -1 per time step
- Undiscounted ($\gamma = 1$)
- Actions: N, E, S, W
- State: current location

# Value Function



# Value Function

- The value function has a recursive formula

$$
\begin{aligned}
V^\pi(s) &= \mathop{\mathbb{E}}_{a_t, a_{t+i}, s_{t+i}} \left[ \sum_{i=0}^{\infty} \gamma^i r_{t+i} | s_t = s \right] \\
&= \mathop{\mathbb{E}}_{a_t} [r_t | s_t = s] + \gamma \mathop{\mathbb{E}}_{a_t, a_{t+i}, s_{t+i}} \left[ \sum_{i=1}^{\infty} \gamma^i r_{t+i+1} | s_t = s \right] \\
&= \mathop{\mathbb{E}}_{a_t} [r_t | s_t = s] + \gamma \mathop{\mathbb{E}}_{s_{t+1}} [V^\pi(s_{t+1}) | s_t = s] \\
&= \sum_{a,r} P^\pi(a|s_t) p(r|a, s_t) \cdot r + \gamma \sum_{a,s'} P^\pi(a|s_t) p(s'|a, s_t) \cdot V^\pi(s')
\end{aligned}
$$

# Action-Value Function

- Can we use a value function to choose actions?

$$\arg\max_{a} r(s_t, a) + \gamma \mathbb{E}_{p(s_{t+1} \mid s_t, a_t)}[V^{\pi}(s_{t+1})]$$

# Action-Value Function

- Can we use a value function to choose actions?

$$\arg\max_{a} r(s_t, a) + \gamma \mathbb{E}_{p(s_{t+1} \mid s_t, a_t)}[V^{\pi}(s_{t+1})]$$

- Problem: this requires taking the expectation with respect to the environment's dynamics, which we don't have direct access to!
- Instead learn an action-value function, or Q-function: expected returns if you take action a and then follow your policy

$$Q^{\pi}(s, a) = \mathbb{E}[G_t \mid s_t = s, a_t = a]$$

- Relationship:

$$V^{\pi}(s) = \sum_{a} \pi(a \mid s) Q^{\pi}(s, a)$$

- Optimal action:

$$\arg\max_{a} Q^{\pi}(s, a)$$

# Bellman Equation

- The Bellman Equation is a recursive formula for the action-value function:

$$Q^\pi(\mathrm{s}, \mathrm{a}) = r(\mathrm{s}, \mathrm{a}) + \gamma \mathbb{E}_{p(\mathrm{s}' \mid \mathrm{s}, \mathrm{a})\, \pi(\mathrm{a}' \mid \mathrm{s}')}[Q^\pi(\mathrm{s}', \mathrm{a}')]$$

- There are various Bellman equations, and most RL algorithms are based on repeatedly applying one of them.

# Optimal Bellman Equation

- The optimal policy $\pi^*$ is the one that maximizes the expected discounted return, and the optimal action-value function $Q^*$ is the action-value function for $\pi^*$.
- The Optimal Bellman Equation gives a recursive formula for $Q^*$:

$$Q^*(\mathsf{s}, \mathsf{a}) = r(\mathsf{s}, \mathsf{a}) + \gamma \mathbb{E}_{p(\mathsf{s}' \mid \mathsf{s}, \mathsf{a})} \left[ \max_{\mathsf{a}'} Q^*(\mathsf{s}_{t+1}, \mathsf{a}') \,|\, \mathsf{s}_t = \mathsf{s}, \mathsf{a}_t = \mathsf{a} \right]$$

- This system of equations characterizes the optimal action-value function. So maybe we can approximate $Q^*$ by trying to solve the optimal Bellman equation!

# Q-Learning

- Let $Q$ be an action-value function which hopefully approximates $Q^*$.
- The Bellman error is the update to our expected return when we observe the next state $s'$.

$$\underbrace{r(s_t, a_t) + \gamma \max_a Q(s_{t+1}, a)}_{\text{inside } \mathbb{E} \text{ in RHS of Bellman eqn}} - Q(s_t, a_t)$$

- The Bellman equation says the Bellman error is 0 at convergence.
- Q-learning is an algorithm that repeatedly adjusts $Q$ to minimize the Bellman error
- Each time we sample consecutive states and actions $(s_t, a_t, s_{t+1})$:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \underbrace{\left[ r(s_t, a_t) + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]}_{\text{Bellman error}}$$

# Exploration-Exploitation Tradeoff

- Notice: Q-learning only learns about the states and actions it visits.
- Exploration-exploitation tradeoff: the agent should sometimes pick suboptimal actions in order to visit new states and actions.
- Simple solution: $\epsilon$-greedy policy
  - With probability $1 - \epsilon$, choose the optimal action according to $Q$
  - With probability $\epsilon$, choose a random action
- Believe it or not, $\epsilon$-greedy is still used today!

# Q-Learning

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize $S$
    Repeat (for each step of episode):
        Choose $A$ from $S$ using policy derived from $Q$ (e.g., $\varepsilon$-greedy)
        Take action $A$, observe $R$, $S'$
        $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
        $S \leftarrow S'$;
    until $S$ is terminal

# Function Approximation

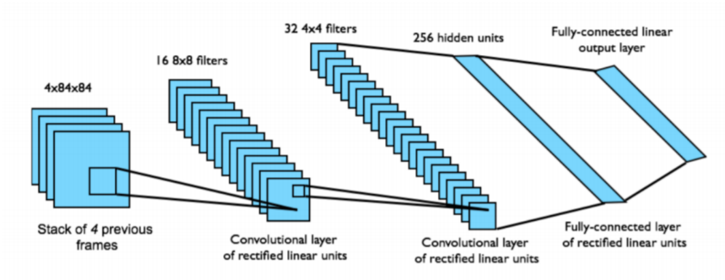
- So far, we've been assuming a tabular representation of $Q$: one entry for every state/action pair.
- This is impractical to store for all but the simplest problems, and doesn't share structure between related states.
- Solution: approximate $Q$ using a parameterized function, e.g.
  - linear function approximation: $Q(\mathrm{s}, \mathrm{a}) = \mathbf{w}^\top \psi(\mathrm{s}, \mathrm{a})$
  - compute $Q$ with a neural net
- Update $Q$ using backprop:

$$t \leftarrow r(\mathrm{s}_t, \mathrm{a}_t) + \gamma \max_{\mathrm{a}} Q(\mathrm{s}_{t+1}, \mathrm{a})$$

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha(t - Q(\mathrm{s}, \mathrm{a})) \frac{\partial Q}{\partial \boldsymbol{\theta}}$$

# Function Approximation with Neural Networks

- Approximating $Q$ with a neural net is a decades-old idea, but DeepMind got it to work really well on Atari games in 2013 ("deep Q-learning")
- They used a very small network by today's standards




- Main technical innovation: store experience into a replay buffer, and perform Q-learning using stored experience
  - Gains sample efficiency by separating environment interaction from optimization — don't need new experience for every SGD update!

# Atari

- Mnih et al., *Nature* 2015. Human-level control through deep reinforcement learning
- Network was given raw pixels as observations
- Same architecture shared between all games
- Assume fully observable environment, even though that's not the case
- After about a day of training on a particular game, often beat "human-level" performance (number of points within 5 minutes of play)
    - Did very well on reactive games, poorly on ones that require planning (e.g. Montezuma's Revenge)
- `https://www.youtube.com/watch?v=V1eYniJ0Rnk`
- `https://www.youtube.com/watch?v=4MlZncshy1Q`

# Wireheading

- If rats have a lever that causes an electrode to stimulate certain “reward centers” in their brain, they’ll keep pressing the lever at the expense of sleep, food, etc.
- RL algorithms show this “wireheading” behavior if the reward function isn’t designed carefully
- `https://blog.openai.com/faulty-reward-functions/`

# Policy Gradient vs. Q-Learning

- Policy gradient and Q-learning use two very different choices of representation: policies and value functions
- Advantage of both methods: don't need to model the environment
- Pros/cons of policy gradient
  - Pro: unbiased estimate of gradient of expected return
  - Pro: can handle a large space of actions (since you only need to sample one)
  - Con: high variance updates (implies poor sample efficiency)
  - Con: doesn't do credit assignment
- Pros/cons of Q-learning
  - Pro: lower variance updates, more sample efficient
  - Pro: does credit assignment
  - Con: biased updates since Q function is approximate (drinks its own Kool-Aid)
  - Con: hard to handle many actions (since you need to take the max)

# AlphaGo

- Most of the problem domains we've discussed so far were natural application areas for deep learning (e.g. vision, language)
  - We know they can be done on a neural architecture (i.e. the human brain)
  - The predictions are inherently ambiguous, so we need to find statistical structure
- Board games are a classic AI domain which relied heavily on sophisticated search techniques with a little bit of machine learning
  - Full observations, deterministic environment — why would we need uncertainty?
- The second part of the lecture is about AlphaGo, DeepMind's Go playing system which took the world by storm in 2016 by defeating the human Go champion Lee Sedol
- Combines ideas from our last two lectures (policy gradient and value function learning)

# AlphaGo

Some milestones in computer game playing:

- 1949 — Claude Shannon proposes the idea of game tree search, explaining how games could be solved algorithmically in principle
- 1951 — Alan Turing writes a chess program that he executes by hand
- 1956 — Arthur Samuel writes a program that plays checkers better than he does
- 1968 — An algorithm defeats human novices at Go

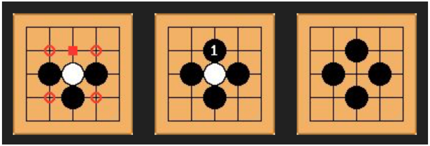
...silence...

- 1992 — TD-Gammon plays backgammon competitively with the best human players
- 1996 — Chinook wins the US National Checkers Championship
- 1997 — DeepBlue defeats world chess champion Garry Kasparov

After chess, Go was humanity's last stand

# Go

- Played on a $19 \times 19$ board
- Two players, black and white, each place one stone per turn
- Capture opponent's stones by surrounding them



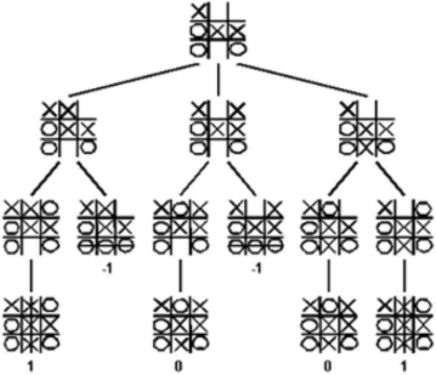
# Go

What makes Go so challenging:

- Hundreds of legal moves from any position, many of which are plausible
- Games can last hundreds of moves
- Unlike Chess, endgames are too complicated to solve exactly (endgames had been a major strength of computer players for games like Chess)
- Heavily dependent on pattern recognition

# Game Trees

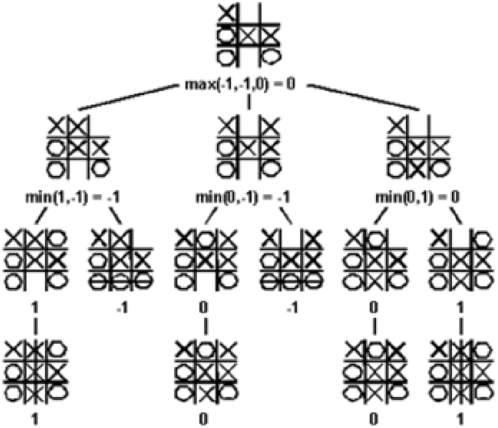
- Each node corresponds to a legal state of the game.
- The children of a node correspond to possible actions taken by a player.
- Leaf nodes are ones where we can compute the value since a win/draw condition was met




https://www.cs.cmu.edu/~adamchik/15-121/lectures/Game%20Trees/Game%20Trees.html

# Game Trees

- To label the internal nodes, take the max over the children if it's Player 1's turn, min over the children if it's Player 2's turn




https://www.cs.cmu.edu/~adamchik/15-121/lectures/Game%20Trees/Game%20Trees.html

# Game Trees

- As Claude Shannon pointed out in 1949, for games with finite numbers of states, you can solve them in principle by drawing out the whole game tree.
- Ways to deal with the exponential blowup
  - Search to some fixed depth, and then estimate the value using an evaluation function
  - Prioritize exploring the most promising actions for each player (according to the evaluation function)
- Having a good evaluation function is key to good performance
  - Traditionally, this was the main application of machine learning to game playing
  - For programs like Deep Blue, the evaluation function would be a learned linear function of carefully hand-designed features

Now for DeepMind's computer Go player, AlphaGo...

# Supervised Learning to Predict Expert Moves

- Can a computer play Go without any search?

# Supervised Learning to Predict Expert Moves

- Can a computer play Go without any search?
- **Input:** a $19 \times 19$ ternary (black/white/empty) image — about half the size of MNIST!
- **Prediction:** a distribution over all (legal) next moves
- **Training data:** KGS Go Server, consisting of 160,000 games and 29 million board/next-move pairs
- **Architecture:** fairly generic conv net
- When playing for real, choose the highest-probability move rather than sampling from the distribution
- This network, which just predicted expert moves, could beat a fairly strong program called GnuGo 97% of the time.
    - This was amazing — basically all strong game players had been based on some sort of search over the game tree

# Self-Play and REINFORCE

- The problem from training with expert data: there are only 160,000 games in the database. What if we overfit?
- There is effecitvely infinite data from self-play
  - Have the network repeatedly play against itself as its opponent
  - For stability, it should also play against older versions of itself
- Start with the policy which samples from the predictive distribution over expert moves
  - The network which computes the policy is called the policy network
- REINFORCE algorithm: update the policy to maximize the expected reward $r$ at the end of the game (in this case, $r = +1$ for win, $-1$ for loss)
- If $\boldsymbol{\theta}$ denotes the parameters of the policy network, $a_t$ is the action at time t, and $s_t$ is the state of the board, and $z$ the rollout of the rest of the game using the current policy

$$R = \mathbb{E}_{a_t \sim p_{\boldsymbol{\theta}}(a_t \mid s_t)}[\mathbb{E}[r(z) \mid s_t, a_t]]$$

# Self-Play and REINFORCE

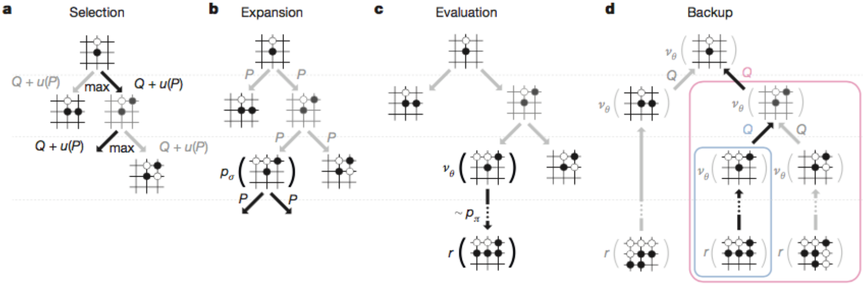
- Gradient of the expected reward:

$$\begin{aligned}
\frac{\partial R}{\partial \boldsymbol{\theta}} &= \frac{\partial R}{\partial \boldsymbol{\theta}} \mathbb{E}_{a_t \sim p_{\boldsymbol{\theta}}(a_t \mid s_t)}[\mathbb{E}[r(z) \mid s_t, a_t]] \\
&= \frac{\partial}{\partial \boldsymbol{\theta}} \sum_{a_t} \sum_{z} p_{\boldsymbol{\theta}}(a_t \mid s_t) p(z|s_t, a_t) R(z) \\
&= \sum_{a_t} \sum_{z} p(z) R(z) \frac{\partial}{\partial \boldsymbol{\theta}} p_{\boldsymbol{\theta}}(a_t \mid s_t) \\
&= \sum_{a_t} \sum_{z} p(z \mid s_t, a_t) R(z) p_{\boldsymbol{\theta}}(a_t \mid s_t) \frac{\partial}{\partial \boldsymbol{\theta}} \log p_{\boldsymbol{\theta}}(a_t \mid s_t) \\
&= \mathbb{E}_{p_{\boldsymbol{\theta}}(a_t \mid s_t)} \left[ \mathbb{E}_{p(z \mid s_t, a_t)} \left[ R(z) \frac{\partial}{\partial \boldsymbol{\theta}} \log p_{\boldsymbol{\theta}}(a_t \mid s_t) \right] \right]
\end{aligned}$$

- English translation: sample the action from the policy, then sample the rollout for the rest of the game.
  - If you win, update the parameters to make the action more likely. If you lose, update them to make it less likely.

# Monte Carlo Tree Search

- In 2006, computer Go was revolutionized by a technique called Monte Carlo Tree Search.




Silver et al., 2016

- Estimate the value of a position by simulating lots of rollouts, i.e. games played randomly using a quick-and-dirty policy
- Keep track of number of wins and losses for each node in the tree
- Key question: how to select which parts of the tree to evaluate?

# Monte Carlo Tree Search

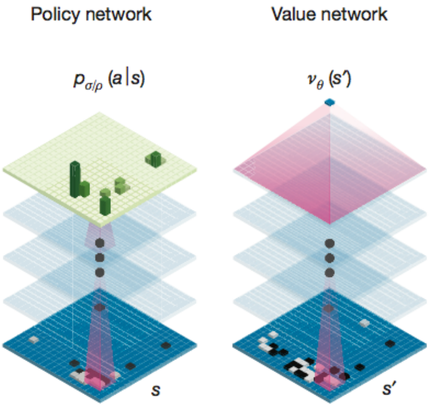
- The selection step determines which part of the game tree to spend computational resources on simulating.
- This is an instance of the exploration-exploitation
  - Want to focus on good actions for the current player
  - But want to explore parts of the tree we're still uncertain about
- Uniform Confidence Bound (UCB) is a common heuristic; choose the node which has the largest frequentist upper confidence bound on its value:

$$\mu_i + \sqrt{\frac{2 \log N}{N_i}}$$

- $\mu_i$ = fraction of wins for action $i$, $N_i$ = number of times we've tried action $i$, $N$ = total times we've visited this node

# Tree Search and Value Networks

- We just saw the policy network. But AlphaGo also has another network called a value network.
- This network tries to predict, for a given position, which player has the advantage.
- This is just a vanilla conv net trained with least-squares regression.
- Data comes from the board positions and outcomes encountered during self-play.




Silver et al., 2016

# Policy and Value Networks

- AlphaGo combined the policy and value networks with Monte Carlo Tree Search
- Policy network used to simulate rollouts
- Value network used to evaluate leaf positions

# AlphaGo Timeline

- **Summer 2014** — start of the project (internship project for UofT grad student Chris Maddison)
- **October 2015** — AlphaGo defeats European champion
  - First time a computer Go player defeated a human professional without handicap — previously believed to be a decade away
- **January 2016** — publication of Nature article "Mastering the game of Go with deep neural networks and tree search"
- **March 2016** — AlphaGo defeats gradmaster Lee Sedol
- **October 2017** — AlphaGo Zero far surpasses the original AlphaGo without training on any human data
- **Decemter 2017** — it beats the best chess programs too, for good measure

# AlphaGo

- Most of the Go world expected AlphaGo to lose 5-0 (even after it had beaten the European champion)
- It won the match 4-1
- Some of its moves seemed bizarre to human experts, but turned out to be really good
- Its one loss occurred when Lee Sedol played a move unlike anything in the training data

# AlphaGo

Further reading:

- Silver et al., 2016. Mastering the game of Go with deep neural networks and tree search. *Nature* http://www.nature.com/nature/journal/v529/n7587/full/nature16961.html
- Scientific American: https://www.scientificamerican.com/article/how-the-computer-beat-the-go-master/
- Talk by the DeepMind CEO: https://www.youtube.com/watch?v=aiwQsa_7ZIQ&list=PLqYmG7hTraZCGIymT8wVVIXLWkKPNBoFN&index=8